Name::

1 Looking at the Behavior of Numbers

Zero, negative numbers, and algebra have not always existed. Actually, even counting numbers have not always existed. For this activity we will be contemplating the time where counting numbers were in use and where the huge problem of considering zero as a number itself was being debated.

To consider the behavior of numbers, we will be breaking down what we know about counting numbers(also known as natural numbers.) Follow along and fill in the blanks of the worksheet.**Begin with the left column first.** Continue working through the left column onto the next page before working through the right column. As you work the right column, compare what you did on the left column.

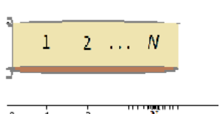
1. Add one number to another number.
   Rule: It changes.
   Example $1 + 1 = 2$ or $2 + 2 = 4$

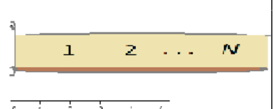
3. Axiom of Archimedes
   Rule: Add something to itself enough times and it will excede the magnitude of any other number.
   Example $2_1 + 2_2 + ... + 2_n > y, \forall y \in N \ni n$ is sufficiently large.

5. Add one number to another
   Rule(Think of another rule, other than what is in 1): ______________________________

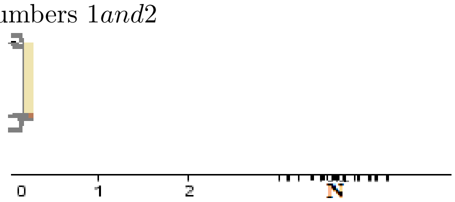
7. Subtract one number from another
   Rule:______________________________

9. Now, Think of multiplying of stretching numbers. The "rubber band" idea below will help illustrate this for a multiplication of 2 of all natural numbers on the number line.

   rubberband.png 

   rubberband.png 

11. Multiplying the all natural numbers on the number line by $\frac{1}{2}$. What happens to the rubber band?
   ____
   Complete the picture for multiplying by $\frac{1}{2}$. Don't forget to include the numbers $1 and 2$

   the rubberband.png 

2. Add Zero to itself and you get Zero. What happens?...It stays the same.

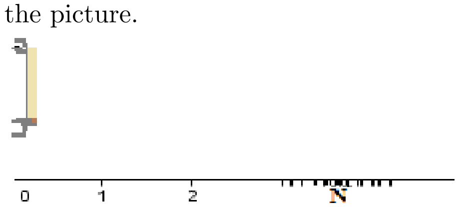
4. Consider Zero in Archimedes Axiom. What happens? Zero refuses to get bigger.

6. Add zero to any other number. What happens? ______________________________

8. Subtract Zero from any other number. What happens? It again stays the same.

10. ☻☹☺

12. What happens to the rubber band when you multiply by zero? Complete the picture.

   the rubberband.png 

   Zero has colapsed the entire number line☹

13. Given that a number n is multiplied by 2, so we have $2n$, how would you undo it to get the number n back? ____________________________________

14. We know that any number, n, times Zero, equals Zero. $n \times 0 = 0$. So then, given Zero, how would you undo the multiplication to get the number n back? ____________________________________

15. An example would be Starting with the number 2 and multiplying it by 3. Giving us $2 \times 3 = 6$. We now have 6 but we want the number 2 back, so we divide by 3. $\frac{6}{3} = 2$☺ Rule: Dividing will undo a multiplication, giving you back the number you started with.

16. If this was true, then $\frac{(2\times0)}{0} = 2$, $\frac{(3\times0)}{0} = 3$, and $\frac{(4\times0)}{0} = 4$. But, we know that all of these can be written as $\frac{0}{0}$. So then does $\frac{0}{0} = 2, 3, and 4$, all at the same time? As we have been disusing, a fraction is just a representation, or name, of a specific number. Saying a specific number equals 2,3, and 4, is more than a little contradictory.

17. If today, we only were using the counting numbers, What would life be like? If we were going to count zero as a number itself, how come I didn't start this list with 0 ?

18. Now consider $\frac{1}{0} \times 0$. Shouldn't the expression reduce to 1? But as we have seen before, anything times Zero, is Zero.

Lets consider a mind blowing proof

Let $a = 1$ and $b = 1$. Since a and b are equal, it would imply that $b^2 = ab$

Now consider the equation

$$a^2 = a^2 \tag{1}$$

Obviously true since anything is equal to itself. But now, subtract 1 from both sides.

$$a^2 - 1 = a^2 - 1. \tag{2}$$

Remember though, that $a = 1$ and $b = 1 \Rightarrow b^2 = 1$ and $ab = 1$

Therefore we can safely substitute b^2 and $ab = 1$ for the different 1's in equation (3) giving us

$$a^2 - b^2 = a^2 - ab. \tag{3}$$

Factoring the both sides of the equation gives us

$$(a + b)(a - b) = a(a - b). \tag{4}$$

Now Divide both sides of the equation by $(a - b)$

$$\frac{(a + b)(a - b) = a(a - b)}{(a - b)} \Rightarrow (a + b) = a. \tag{5}$$

Seeing something fishy here? Subtract both a from both sides of the equation, and we have now proven that

$$b = 0 \Rightarrow 1 = 0 \tag{6}$$

What!!!!!!!!!! $1 = 0$ So what does this mean? So by multiplying both sides of the equation by 32 we can get $32 = 0$ but since $1 = 0$, we now have $1 = 0 = 32 \Rightarrow 1 = 32$. Is your whole concept of the world of mathematics blowing up??? Does this feel like the work of the devil or something?? It even gets worse if you consider adding units to the equation. I have 1 head but that means I have no head since $1 = 0$. Ok so I have no head. I still have 2 legs, but since we could prove that $7 = 0$, I now have no head and 7 legs.☹ ☹

●Identify and Circle the step you think is causing the problem●

[0]Principles and ideas expressed in this worksheet come from the book *Zero : The Biography of a Dangerous Idea* by CHarles Seife. Mainly from pp.20-23, but also from appendix A, for the proof.